# Package: einsum (via r-universe)

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Type Package

| Title Einstein Summation  |
|---|
| Version 0.1.2   |
| Description The summation notation suggested by Einstein (1916) <doi:10.1002 andp.19163540702=""> is a concise mathematical notation that implicitly sums over repeated indices of n-dimensional arrays. Many ordinary matrix operations (e.g. transpose, matrix multiplication, scalar product, 'diag()', trace etc.) can be written using Einstein notation. The notation is particularly convenient for expressing operations on arrays with more than two dimensions because the respective operators ('tensor products') might not have a standardized name.</doi:10.1002> |
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#### **Description**

Einstein summation is a convenient and concise notation for operations on n-dimensional arrays.

### Usage

```
einsum(equation_string, ...)
einsum_generator(equation_string, compile_function = TRUE)
```

#### **Arguments**

equation\_string

a string in Einstein notation where arrays are separated by ',' and the result is separated by '->'. For example "ij,jk->ik" corresponds to a standard matrix multiplication. Whitespace inside the equation\_string is ignored. Unlike the equivalent functions in Python, einsum() only supports the explicit mode. This means that the equation\_string must contain '->'.

... the arrays that are combined. All arguments are converted to arrays with as.array. compile\_function

boolean that decides if einsum\_generator() returns the result of Rcpp::cppFunction() or the program as a string. Default: TRUE.

#### **Details**

The following table show, how the Einstein notation abbreviates complex summation for arrays/matrices:

| equation_string | Formula   |                         |
|-----------------|---|-------------------------|
| "ij,jk->ik"     | $Y_{ik} = \sum_{j} A_{ij} B_{jk}$               | Matrix multiplication   |
| "ij->ji"'       | $Y = A^T$                                       | Transpose               |
| "ii->i"         | $y = \operatorname{diag}(A)$                    | Diagonal                |
| "ii->ii"        | $Y = \operatorname{diag}(A)I$                   | Diagonal times Identity |
| "ii->"          | $y = \operatorname{trace}(A) = \sum_{i} A_{ii}$ | Trace                   |
| "ijk,mjj->i"    | $y_i = \sum_j \sum_k \sum_m A_{ijk} B_{mjj}$    | Complex 3D operation    |

The function and the conventions are inspired by the einsum() function in NumPy (documentation). Unlike NumPy, 'einsum' only supports the explicit mode. The explicit mode is more flexible and can avoid confusion. The common summary of the Einstein summation to "sum over duplicated indices" however is not a good mental model. A better rule of thumb is "sum over all indices not in the result".

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*Note:* einsum() internally uses C++ code to provide results quickly, the repeated parsing of the equation\_string comes with some overhead. Thus, if you need to do the same calculation over and over again it can be worth to use einsum\_generator() and call the returned the function. einsum\_generator() generates efficient C++ code that can be one or two orders of magnitude faster than einsum().

#### Value

The einsum() function returns an array with one dimension for each index in the result of the equation\_string. For example "ij,jk->ik" produces a 2-dimensional array, "abc,cd,de->abe" produces a 3-dimensional array.

The einsum\_generator() function returns a function that takes one array for each comma-separated input in the equation\_string and returns the same result as einsum(). Or if compile\_function = FALSE, einsum\_generator() function returns a string with the C++ code for such a function.

#### **Examples**

```
mat1 \leftarrow matrix(rnorm(n = 4 * 8), nrow = 4, ncol = 8)
mat2 \leftarrow matrix(rnorm(n = 8 * 3), nrow = 8, ncol = 3)
# Matrix Multiply
mat1 %*% mat2
einsum("ij,jk -> ik", mat1, mat2)
# einsum_generator() works just like einsum() but returns a performant function
mat_mult <- einsum_generator("ij,jk -> ik")
mat_mult(mat1, mat2)
# Diag
mat_sq \leftarrow matrix(rnorm(n = 4 * 4), nrow = 4, ncol = 4)
diag(mat_sq)
einsum("ii->i", mat_sq)
einsum("ii->ii", mat_sq)
# Trace
sum(diag(mat_sq))
einsum("ii->", mat_sq)
# Scalar product
mat3 \leftarrow matrix(rnorm(n = 4 * 8), nrow = 4, ncol = 8)
mat3 * mat1
einsum("ij,ij->ij", mat3, mat1)
# Transpose
t(mat1)
einsum("ij->ji", mat1)
# Batched L2 norm
arr1 <- array(c(mat1, mat3), dim = c(dim(mat1), 2))</pre>
```

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```
c(sum(mat1^2), sum(mat3^2))
einsum("ijb,ijb->b", arr1, arr1)
```

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Einstein Summation

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